Non-Semisimple Representations of Degenerate Double Affine Hecke Algebras

*Sarah Byers (130285984) Supervisor: Dr Martina Balagović

School of Mathematics and Statistics

School of Mathematics & Statistics, Newcastle University

Motivating Question

Is it possible to realise all category $\mathcal{O}$ representations of degenerate double affine Hecke algebras as submodules of objects that are better understood, namely Verma Modules?

Objectives

1. Gain a working knowledge of group theory, representation theory and degenerate double affine Hecke algebras (DAHA)
2. Find all unique periodic Cherednik diagrams that parameterize irreducible representation of degenerate DAHA of 2 and 3 boxes
3. Explicitly describe all irreducible representations corresponding to all diagrams from 2)
4. Using 3) answer the Motivating question for those cases

Introduction

- The study of double affine Hecke Algebras originated from a branches of theoretical physics and mathematics
- Study of their representations is a very active area of research in the UK and abroad
- For AHA, $\dot{H}(n)$, and DAHA, $\ddot{H}_\kappa(n)$, the answer to the motivation question has been answered in literature as follows:

	$\dot{H}(n)$	$\ddot{H}_\kappa(n)$
Semisimple	[GNP]	[B]
Non-semisimple	[GNP]	?

The aim of this project was to find an answer to the lower right corner for the cases n=2,3. This will be done by explicitly calculating the irreducible representations of $\ddot{H}_\kappa(2)$ and $\ddot{H}_\kappa(3)$ for all κ.

Young Diagrams & Their Generalisations (Objective 1)

- The symmetric group S_n, is a well understood group
- A classical result states that all irreducible representations of S_n are in 1-to-1 correspondence with objects called Young diagrams (Fig. 1a)
- For AHA and DAHA, a similar correspondence exists with generalisations of Young diagrams
- Non-semisimple representations of degenerate DAHA share this relation with Periodic Cherednik diagrams (Fig. 1b)

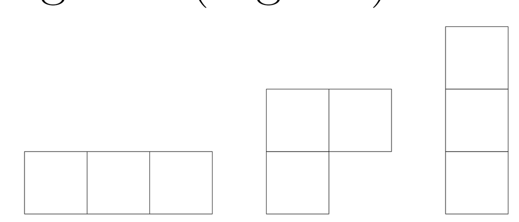


Fig. 1a. Young diagrams n=3

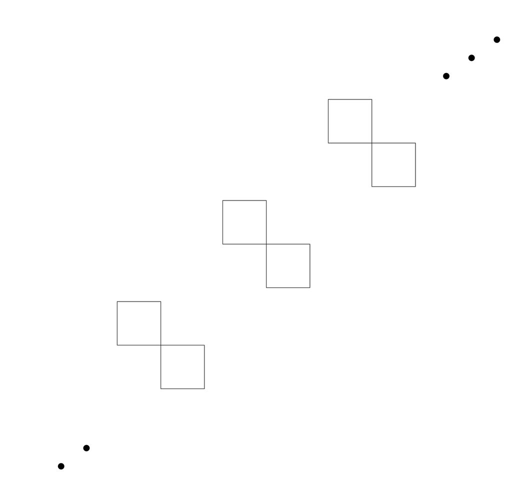


Fig. 1b. Periodic Cherednik diagram for $\ddot{H}_4(2)$

Classifying Diagrams (Objective 2)

Lemma: If D′ is a translation of D, the answer to the motivating question is the same for $L_{D'}$ and L_D.

- Diagrams for n=2

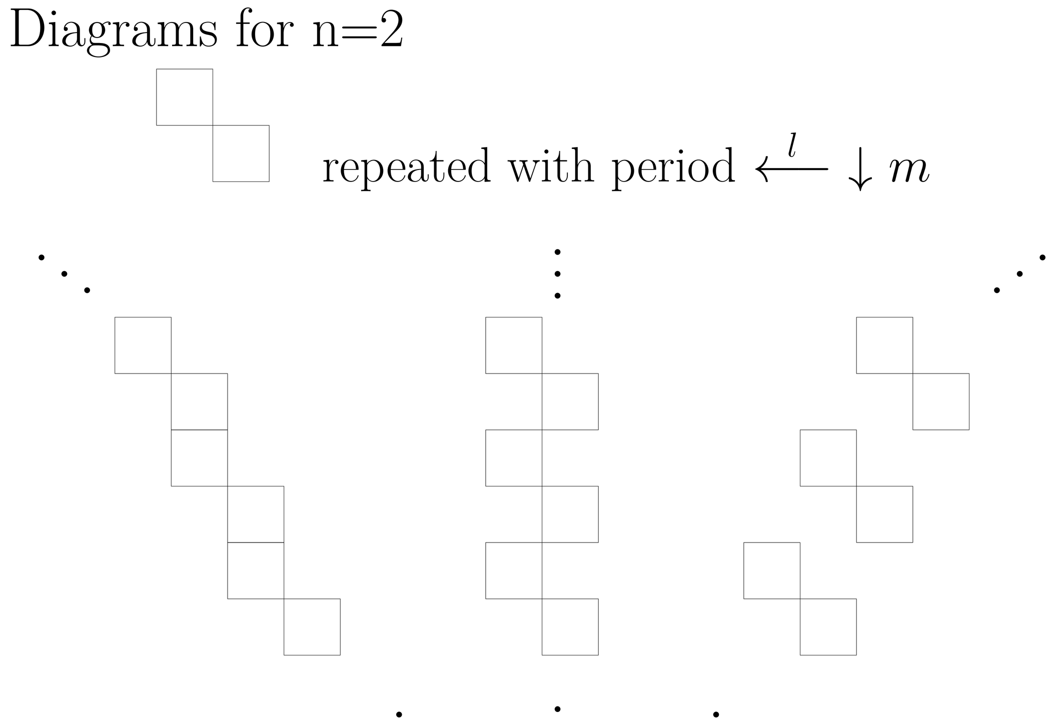
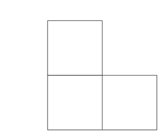
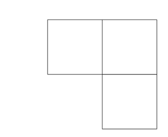


Fig. 2. Periodic Cherednik diagrams for n=2; l=-1, l=0 and l=1

- Diagrams for n=3, m=2

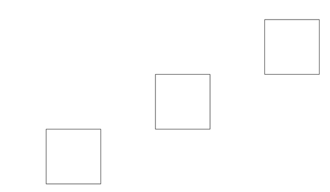


Repeated with period l≥-1



Repeated with period l ≥ 0

- Diagrams for n=3, m=3



With gaps of $k_1 \geq -1$ and $k_2 \geq -1$ between blocks
Repeated with period $l \geq k_1+k_2+1$

Verma Modules

To each diagram D we can associate a representation M_D (Verma module). Verma modules are well understood, in terms of both their structure and their interactions with the algebra (Fig. 3).

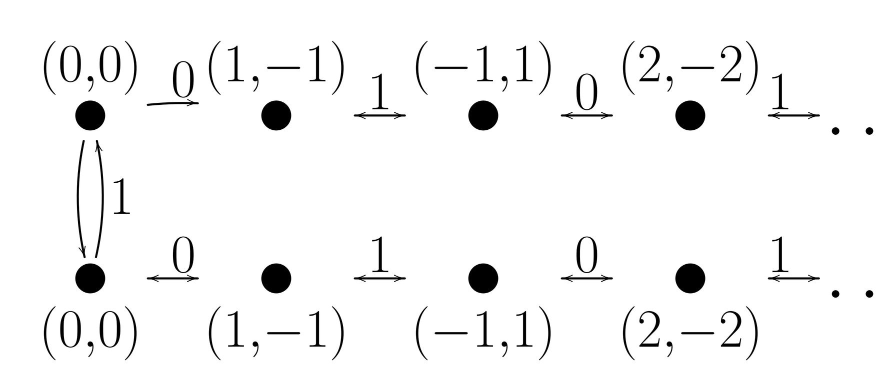


Fig. 3. Graphical Representation of M_D for $\ddot{H}_1(2)$

Irreducible modules (Objective 3)

- An irreducible module is a building block of any representation, they can not be further decomposed and any representation can be formed from them
- Associated with each of our diagrams D is the irreducible module L_D; it is formed by the construction called quotient on M_D but this can be complicated to realise. So instead we look to find L_D using a simpler construction called submodule.

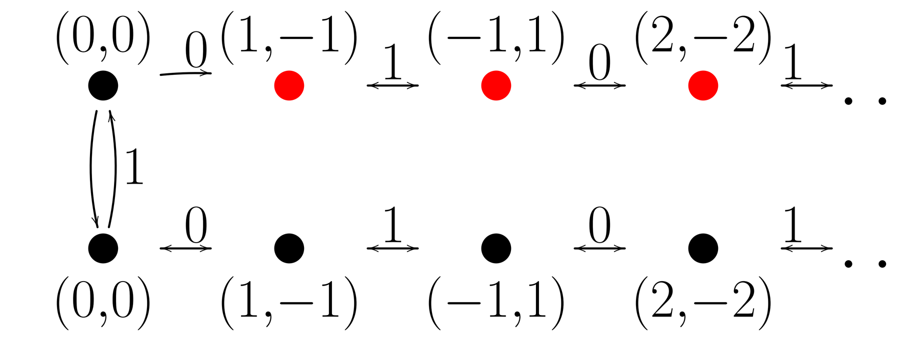


Fig. 4. Graphical Representation of L_D (in black) for $\ddot{H}_1(2)$

Results (Objective 4)

Following similar calculations for all κ we were able to establish:

Theorem 1

Let n=2, D any periodic Cherednik diagram which is not a skew Young diagram. Then there exists a sequence of integers μ, constructed from D, such that L_D embeds into M_μ, $L_D \hookrightarrow M_\mu$

Theorem 2

Let n=3, D any periodic Cherednik diagram which is not a skew Young diagram, with 3 boxes organised in 2 rows. Then there exists a sequence of integers μ, constructed from D, such that L_D embeds into M_μ, $L_D \hookrightarrow M_\mu$

Conclusions

1. All irreducible representations of degenerate DAHA with n=2 can be realised as submodules of the better understood Verma modules
2. The same is true for irreducible representations of degenerate DAHA with n=3 and m=2
3. It is our conjecture that all non-semisimple representations of degenerate DAHA can be realised as submodules of the better understood Verma modules

Our conjecture is in contrast to the semisimple case, in which the answer is yes to all but a few exceptional diagrams. It would be interesting to have the answer to our Motivating Question for all n.

[OV] A.M.Vershik, A.Yu.Okounkov, *A New Approach to the Representation Theory of the Symmetric Groups, Selecta Math.* (New Series) V.2(1996)
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[S] T. Suzuki, *Classification of Simple modules over degenerate double affine Hecke Algebras of type A*, Int. Math. Res. Not. 2005, no.27, 1621-1656